EFFECTIVE EMISSIVITY OF THE ENDFACE OF A PACKET OF SCREEN-VACUUM THERMAL INSULATION

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An analytic expression for the effective emissivity of an SVTI packet is derived with the help of the quasi-diffusion approximation [1] under the assumption that the reflection at the screens is diffusely specular.

The development and extensive use of screen-vacuum thermal insulation (SVTI) has significantly stimulated progress in cryogenic technology. Even now, however, there are unutilized reserves in this area. It is well known that the effective thermal conductivity of SVTI achieved in calorimetric setups is 3-5 times lower than the value that can be obtained for real cryogenic equipment. One of the main reasons for the degradation of the quality of SVTI in the parts is the presence of technological gaps between the packets or between a packet and thermal bridges. The significant change in the efficiency of SVTI is also linked with the existence of heat transfer between the end of its packet and the drainage necks or other constructional elements. To describe all these problems it is necessary to have data on the emissivity of the endface of the SVTI packet, which cannot yet be determined experimentally. In this paper we propose a method for calculating this parameter.

In practice the endface of an SVTI packet consists of a collection of separate gaps between SVTI screens, whose length ℓ is much greater than the width h, i.e., the distance between the screens. A method for calculating numerically the emissivity of not very long gaps, containing diffusely reflecting walls, was developed by Sperrou [2]. In [2] the emissivities of flat channels with the emissivity of the walls $\varepsilon \geq 0.5$ were evaluated, while in [3] they were evaluated for $0.01 \leq \varepsilon \leq 0.5$, i.e., for SVTI packets. For sufficiently long channels the numerical solution of this problem presents significant difficulties. There are also no estimates of the effect of the specularity of the reflecting surfaces on the parameter under study, which to one extent or another happens in real materials.

The purpose of this work is to derive analytic relations for calculating the effective emissivity of an SVTI packet. On the basis of the problem formulated the packet can be regarded as a collection of separate flat channels, making the assumption that the temperature remains constant along each channel. In SVTI the temperature of neighboring screens is virtually identical. For this reason, the temperatures T and the emissivities of both walls of the channel are assumed to be the same and constant along the coordinate x. To simplify the problem and without losing generality we shall model the medium outside the SVTI packet by an absolutely black body whose temperature equals zero.

Consider a gap between parallel flat strips S_1 and S_2 of width ℓ. Each strip is bounded in its plane by the straight lines $x = 0$ and $x = \ell$. The emissivity of the gap is defined as the ratio of the heat-flux density Q_{inc}, incident from the side of the gap on its endface AD (Fig. 1), to the radiation of an absolutely black body:

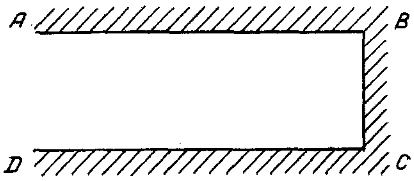

Fig. 1. Transverse section of a flat gap.

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$$\varepsilon_a = \frac{Q_{inc}}{\sigma T^4}. \tag{1}$$

We shall first study the case when the radiation flux is reflected in a specularly diffuse manner by the walls of the gap. In this case we shall confine our analysis to values of ℓ/h that are so large that the effect of the gap at the end BD (Fig. 1) on the emissivity can be neglected. The diffuse part of the effective flux at the point y equals

$$\hat{q}(\mathbf{y}) = \varepsilon\sigma T^4 + q^d(\mathbf{y}).$$

Then the effective flux at the point y in the direction s is expressed in terms of the diffuse component of the effective flux:

$$q(\mathbf{y}, \mathbf{s}) = \frac{1}{\pi}\sum_{k=0}^{\infty}(1-\alpha)^k(1-\varepsilon)^k\hat{q}(\mathbf{y}^{k+1}), \tag{2}$$

where $\mathbf{y}^{k+1}$ is a point obtained as a result of k specular reflections from the point y in the direction $\mathbf{s}_1 = \mathbf{s} - 2\mathbf{n}_y(\mathbf{n}_y, \mathbf{s})$. The function $\hat{q}(x)$ according to [1], satisfies the following integral equation:

$$\hat{q}(x) = \varepsilon\sigma T^4 + \alpha(1-\varepsilon)\left[\frac{h^2}{2\mu}\sum_{k=0}^{\infty}\mu^k k^2\int_0^l \frac{\hat{q}(x')\,dx'}{[h^2k^2 + (x-x')^2]^{3/2}} + F\right], \tag{3}$$

where $\mu = (1-\varepsilon)(1-\alpha)$ and F is the radiation flux from the ends AD and BC arriving at the point x. The radiation flux incident on the lateral surface, averaged over the height of the gap, as shown in [1], has the following form:

$$Q_{inc} = \sum_{k=0}^{\infty}\frac{\mu^k}{h}\int_0^l x\hat{q}(x)\left[(k^2h^2 + x^2)^{-1/2} - ((k+1)^2h^2 + x^2)^{-1/2}\right]dx. \tag{4}$$

Using the quasidiffusion approximation [1] we shall replace the integral equation (3) for the function $\hat{q}(x)$ by a differential equation

$$\frac{d^2\hat{q}}{dx^2} - \varkappa^2\hat{q} = -\varkappa^2(1-\mu)\sigma T^4, \tag{5}$$

where

$$\varkappa^2 = \frac{\varepsilon(1-\alpha)}{\mu(1-\mu)\alpha h^2 P}; \quad P = \frac{1}{2\mu}\sum_{k=1}^{\infty}\mu^k k^2 \ln\frac{l}{kh}.$$

Let $\ell/h \gg 1$. Since the function $\hat{q}(x)$ must be bounded at $x = \ell$ the solution of Eq. (5) can be represented in the form

$$\hat{q}(x) = (1-\mu)\sigma T^4 + c\exp(-\varkappa x). \tag{6}$$

We shall obtain the boundary condition for determining the coefficient c from the integral equation (3), written at the end $x = 0$:

$$\hat{q}(0) = \varepsilon\sigma T^4 + \alpha(1-\varepsilon)\frac{h^2}{2\mu}\sum_{k=0}^{\infty}\mu^k k^2\int_0^l \frac{\hat{q}(x')\,dx'}{[h^2k^2 + x'^2]^{3/2}}. \tag{7}$$

We expand the function $\hat{q}(x')$ in the integrand in a Taylor series around the point $x = 0$. Then up to terms of order $O(h^2/\ell^2)$ we can write

$$\hat{q}(0)\left[1 - \frac{\alpha(1-\varepsilon)}{2\mu}\sum_{k=0}^{\infty}\frac{\mu^k}{\sqrt{\left(\frac{h}{l}\right)^2 k^2 + 1}}\right] = \varepsilon\sigma T^4,$$

whence follows the boundary condition for Eq. (5)

$$\hat{q}(0) = \mu_0\sigma T^4, \tag{8}$$

where

$$\mu_0 = \varepsilon\left[1 - \frac{\alpha(1-\varepsilon)}{2\mu}\sum_{k=0}^{\infty}\frac{\mu^k}{\sqrt{\left(\frac{h}{l}\right)^2 k^2 + 1}}\right]^{-1}.$$

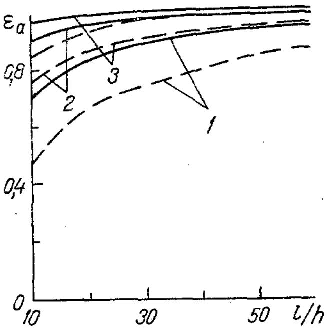

Fig. 2

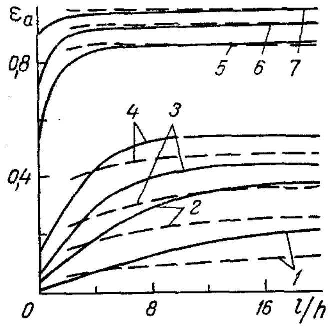

Fig. 3

Fig. 2. Effective emissivity of the endface of the gap versus the relative length of the gap for different emissivities of the walls in the case of purely specular reflection at the walls. The solid lines show the data of [2] and the broken lines show the calculation based on the formula (11): 1) $\varepsilon = 0.1$; 2) 0.3; 3) 0.5.

Fig. 3. Effective emissivity of the endface of the gap versus the relative length of the gap in the case of diffusely reflecting walls for different emissivity of the walls. The solid lines show the data of [2] (curves 5-7) and [3] (curves 1-4); the broken lines show the calculation based on the formula (12): 1) $\varepsilon = 0.01$; 2) 0.03; 3) 0.05; 4) 0.1; 5) 0.5; 6) 0.7; 7) 0.9.

The solution of Eq. (5), satisfying the condition (8) and bounded at infinity, has the form

$$\frac{\hat{q}(x)}{\sigma T^4} = 1 - \mu + (\mu_0 - 1 + \mu)\exp(-\varkappa x). \tag{9}$$

Substituting this expression into (4) and (1) we obtain the following value of the effective emissivity of the endface of the gap:

$$\varepsilon_a = \sum_{k=0}^{\infty} \frac{\mu^k}{h} \int_0^l x\,[1 - \mu + (\mu_0 - 1 + \mu)\exp(-\varkappa x)]\,[(k^2h^2 + x^2)^{-1/2} - ((k+1)^2h^2 + x^2)^{-1/2}]\,dx. \tag{10}$$

In the case of purely specular reflection $\alpha = 0$ and, therefore, $\hat{q} = \varepsilon\sigma T^4$. Then

$$\varepsilon_a = \sum_{k=0}^{\infty} \mu^k \frac{\varepsilon}{h} \int_0^l x\,[(k^2h^2 + x^2)^{-1/2} - ((k+1)^2h^2 + x^2)^{-1/2}]\,dx =$$
$$= 1 - \varepsilon \sum_{k=0}^{\infty} \mu^k \left[\sqrt{(k+1)^2 + \left(\frac{l}{h}\right)^2} - \sqrt{k^2 + \left(\frac{l}{h}\right)^2}\right]. \tag{11}$$

Figure 2 compares the values of ε_a, obtained from the formula (11) and the results of [2]. In the limit $\ell/h \to \infty$ the effect of the closed end approaches zero and the curves in Fig. 2 converge toward one another as ℓ/h increases.

We shall now examine purely diffuse reflection, i.e., $\mu = 0$. In this case it is possible to take into account the arrival of radiation from the closed end BC:

$$\varepsilon_a = \frac{1}{h}\int_0^l \frac{q^r(x')}{\sigma T^4}\left(1 - \frac{x'}{\sqrt{h^2 + x'^2}}\right)dx' + \frac{Q}{\sigma T^4}\,\frac{\sqrt{l^2 + h^2} - l}{h}. \tag{12}$$

Here we take for Q the average flux at the endface of the gap BC:

$$Q = \varepsilon\sigma T^4 + (1 - \varepsilon)\frac{1}{h}\int_0^l q^r(x')\left(1 - \frac{x'}{\sqrt{h^2 + x'^2}}\right)dx'. \tag{13}$$

For $h \ll \ell$, $q^r(x)$ satisfies the differential equation

$$\frac{d^2q^r}{dx^2} - \varkappa_1^2 q^r(x) = -\varkappa_1^2 \sigma T^4, \tag{14}$$

where $\kappa_1^2 = 2\varepsilon/(1-\varepsilon)h_2 \ln(1/h)$. The general solution of Eq. (14) has the form

$$q^r(x) = \sigma T^4 (1 + c_1 \exp(-\varkappa_1 x) + c_2 \exp(\varkappa_1 x)). \tag{15}$$

The boundary conditions for determining the coefficient c_1 and c_2 can be found from the integral equation satisfied by the effective radiation flux density:

$$q^r(x) = \varepsilon\sigma T^4 + (1-\varepsilon)\left[\frac{h^2}{2}\int_0^l \frac{q^r(x')\,dx'}{[h^2+(x-x')^2]^{3/2}} + \frac{Q}{2}\left(1 - \frac{l-x}{\sqrt{h^2+(l-x)^2}}\right)\right]. \tag{16}$$

We write the boundary conditions for Eq. (14):

$$q^r(0) = \varepsilon\sigma T^4 + (1-\varepsilon)\left[\frac{h^2}{2}\int_0^l \frac{q^r(x')\,dx'}{(h^2+x'^2)^{3/2}} + \frac{Q}{2}\left(1 - \frac{l}{\sqrt{h^2+l^2}}\right)\right],$$

$$q^r(l) = \varepsilon\sigma T^4 + (1-\varepsilon)\left[\frac{h^2}{2}\int_0^l \frac{q^r(x')\,dx'}{[h^2+(l-x')^2]^{3/2}} + \frac{Q}{2}\right]. \tag{17}$$

Substituting the expression (15) for $q^r(x)$ into (13) and (17) we obtain a system of two algebraic equations for c_1 and c_2. Having determined $q^r(x)$ we find ε_a using the formula (12).

The following estimate is valid for the absolute error in replacing the integral equation by a differential equation:

$$\delta q^r = o\left(\frac{1-\varepsilon}{\varepsilon}\left(\frac{h}{l}\right)^2 \ln\frac{l}{h}\right) \quad \text{as } h \to 0. \tag{18}$$

The relationship (18) permits drawing the following conclusions: the smaller ε, the larger the value of ℓ/h must be in order to achieve a given accuracy. In application to vacuum insulation, where $0.01 \leq \varepsilon \leq 0.1$, in order to achieve an accuracy of ~5% the simplified estimate $h/\ell = \sqrt{\varepsilon}/10$ can be employed.

Figure 3 compares the values of εa obtained in the case of diffuse reflection with the data of [2, 3]. As one can see from the figure, for $\varepsilon \geq 0.5$ good agreement with the results in [2] is already achieved for $\ell/h \geq 6$. The values of ε_a for $\varepsilon \leq 0.1$ with $\ell/h \leq 20$ are presented for [3]. Because of this the results based on the formula (12) differ from the data of [3], but as ℓ/h increases the curves converge towards one another. The estimate (18) shows that the required accuracy will be achieved with $\ell/h \sim 100$.

NOTATION

h, distance between the screens of the gap; ℓ, length of a screen; Q, average flux at the endface of the gap; q(x), $q^r(x)$, effective radiation fluxes on the screen at the point with the coordinate x in the case of diffusely specular and diffuse reflection, respectively; $q^d(y)$, part of the flux reflected diffusely from the wall at the point y; $\hat{q}(y)$, diffuse part of the effective flux; δq^r, absolute error in determining the effective flux when the integral equation is replaced by a differential equation; T, temperature of the gap walls, α, degree of diffuseness of the reflection $q^d(y) = \alpha q^{inc}(y)$; ε, emissivity of the gap walls; ε_a, effective emissivity of the endface of the gap; and σ, Stefan–Boltzmann constant.

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